$$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} \dots \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots (-1)^{n+1} \frac{x^n}{n}$$

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18. $\sum_{n=2}^{\infty} \left| \frac{1}{\ln n} (-1)^{n+1} \right| > \cancel{\sum \frac{1}{n}}$ ~~diverge~~

?

$\ln(\cancel{n}) < n$

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln n} = \frac{-1}{\ln 2} + \frac{1}{\ln 3} - \frac{1}{\ln 4} + \dots$$

1. signs alternate? ✓
2. |terms| get smaller ✓
3. seq converge to 0 ✓✓

series converges by AST

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22. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right) =$ diverges

$$\int_1^{\infty} \frac{1}{x} - \frac{1}{x^2} \, dx \quad \text{diverges}$$

$$\lim_{b\to\infty} \ln x + \frac{1}{x} \Big|_1^b$$

$$\lim_{b\to\infty} \left(\underset{\infty}{\underset{\downarrow}{\ln b}} + \underset{0}{\underset{\downarrow}{\frac{1}{b}}}\right) - \left(\underset{0}{\underset{\downarrow}{\ln 1}} + \underset{1}{\underset{\downarrow}{1}}\right) = \infty$$

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19 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$ exp / power

n^{th} term test

$$\lim_{n\to\infty} \frac{10^n}{n^{10}} = \lim_{n\to\infty} \frac{10^n \ln 10}{10n^9}$$

$$= \lim_{n\to\infty} \frac{10^n (\ln 10)^2}{10 \cdot 9 \cdot n^8}$$

$$\vdots$$

$$\lim_{n\to\infty} \frac{10^n (\ln 10)^{10}}{10!} = \infty$$

seq does not converge to 0
series diverges

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9.5b Alternating Series, Checking Endpoints

Alternating Series Test with remainder

AST if 1. signs alternate

2. $|a_{n+1}| < |a_n|$

(terms get smaller)

3. $a_n \to 0$

(seq converges to 0)

then series converges

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Remainder

$a_1 - a_2 + a_3 - a_4 \cdots a_n$ | Rem

ϵ

$\epsilon \leq |a_{n+1}|$

Error has the same sign as the next term

Feb 10-8:06 AM

Prove the alternating harmonic series is convergent but not absolutely convergent

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} \quad 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\ \ldots$$

Converges by AST

$$\sum_{n=1}^{\infty}\left|(-1)^{n+1}\frac{1}{n}\right| = \sum_{n=1}^{\infty}\frac{1}{n} \quad \text{diverges}$$

conditional convergence

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Conditional Convergence

$\sum |a_n|$ diverges

$\sum a_n$ converges ← probably prove this with AST

must be alternating series

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Find the interval of convergence for the following series. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{2n}$$

ratio test

$2(n+1)$

$$\lim_{n\to\infty}\left|\frac{x^{2n+2}}{2n+2}\cdot\frac{2n}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{2n}{2n+2}\cdot x^2\right| = x^2$$

$\frac{2n}{2n+2} \to 1$

$x^2 < 1$

$-1 \leq x \leq 1$

ratio test fails at the endpts

$x=1$ $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n} = \frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}\ldots$

converges by AST

$x=-1$ $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n}$ converges by AST

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$$\sum_{n=0}^{\infty}\frac{(10x)^n}{n!}$$

Find the ioc

{ ~~don't forget to check endpts~~ }

$$\lim_{n\to\infty}\left|\frac{(10x)^{\cancel{n+1}}}{(n+1)\cancel{!}}\cdot\frac{\cancel{n!}}{\cancel{(10x)^n}}\right| = 0 \text{ for all } x$$

converges for all x

$(-\infty,\infty)$

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$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2n}$ Find the ioc

$$\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{2n+2}\cdot\frac{2n}{(x-3)^n}\right| = \lim_{n\to\infty}\left|(x-3)\,\frac{2n}{2n+2}\right|$$

$$= |x-3| < 1$$

$$-1 \le x-3 < 1$$

$$2 \le x < 4$$

endpts

$x=2$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{2n} = -\frac{1}{2}+\frac{1}{4}-\frac{1}{6}$$

Conv by AST

$x=4$

$$\sum \frac{1}{2n} = \frac{1}{2}+\frac{1}{4}+\frac{1}{6}$$

diverges

$\frac{1}{2}$ harm

p series

$p=1$

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